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Question Paper Code : 77193

B.E./B.Tech. DEGREE EXAMINATION, APRIL/MAY 2015.

Fourth Semester

Electronics and Communication Engineering

MA 6451 – PROBABILITY AND RANDOM PROCESSES

(Common to Biomedical Engineering, Robotics and Automation Engineering)

(Regulation 2013)

Time : Three hours | Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Show that the function $f(x)=\begin{cases} e^{-x} : x \geq 0 \\ 0 : x < 0 \end{cases}$ is a probability density function of a random variable X.

2. The mean and variance of binomial distribution are 5 and 4. Determine the distribution.

3. Find the value of k, if $f(x,y)=kxye^{-(x^2+y^2)} : x \geq 0,\ y \geq 0$ is to be a joint probability density function.

4. What is the angle between the two regression lines?

5. Give an example of evolutionary random process.

6. Define a semi-random telegraph signal process.

7. State any two properties of cross correlation function.

8. Find the auto correlation function whose spectral density is $S(w)=\begin{cases} \pi, & |w| \leq 1 \\ 0 & \text{otherwise} \end{cases}$.

9. Prove that $Y(t)=2X(t)$ is linear.

10. State the relation between input and output of a linear time invariant system.

PART B — (5 × 16 = 80 marks)

11. (a) (i) A continuous random variable X that can assume any value between $X=2$ and $X=5$ has a probability density function given by $f(x)=k(1+x)$. Find $P(X<4)$. (8)

(ii) If the probability that an applicant for a driver's license will pass the road test on any given trial is 0.8, what is the probability that he will finally pass the test on the 4th trial? Also find the probability that he will finally pass the test in less than 4 trials. (8)

Or

(b) (i) Find the moment generating function of exponential distribution and hence find the mean and variance of exponential distribution. (8)

(ii) If the probability mass function of a random variable X is given by $P[X=x]=kx^3$, $x=1,2,3,4$, find the value of k, $P\left[\left(\frac{1}{2}<X<\frac{5}{2}\right)/X>1\right]$, mean and variance of X. (8)

12. (a) (i) If the joint probability distribution function of a two dimensional random variable (X,Y) is given by $F(x,y)=\begin{cases}(1-e^{-x})(1-e^{-y}): x>0, y>0\\ 0: \text{otherwise}\end{cases}$, find the marginal densities of X and Y. Are X and Y independent? Find $P[1<X<3, 1<Y<2]$. (8)

(ii) Find the coefficient of correlation between X and Y from the data given below. (8)

X:	65	66	67	67	68	69	70	72
Y:	67	68	65	68	72	72	69	71

Or

(b) (i) The two lines of regression are $8X-10Y+66=0$, $40X-18Y-214=0$. The variance of X is 9. Find the mean values of X and Y. Also find the coefficient of correlation between the variables X and Y. (8)

(ii) Two random variables X and Y have the following joint probability density function. $f(x,y)=\begin{cases}x+y: 0\le x\le 1, 0\le y\le 1\\ 0 \quad : \text{otherwise}\end{cases}$. Find the probability density function of the random variable $U=XY$. (8)

13. (a) (i) Show that the process $X(t)=A\cos\lambda t+B\sin\lambda t$ where A and B are random variables, is wide sense stationary process if $E(A)=E(B)=E(AB)=0$, $E(A^2)=E(B^2)$. (8)

(ii) There are 2 white marbles in Urn A and 3 red marbles in Urn B. At each step of the process, a marble is selected from each urn and the 2 marbles selected are interchanged. The state of the related Markov chain is the number of red marbles in Urn A after the interchange. What is the probability that there are 2 red marbles in Urn A after 3 steps? In the long run, what is the probability that there are 2 red marbles in Urn A? (8)

Or

(b) (i) A radioactive source emits particles at a rate of 5 per minute in accordance with Poisson process. Each particle emitted has a probability 0.6 of being recorded. Find the probability that 10 particles are recorded in 4 minute period. (8)

(ii) Check if a random telegraph signal process is wide sense stationary. (8)

14. (a) (i) Consider two random processes $X(t)=3\cos(\omega t+\theta)$ and $Y(t)=2\cos(\omega t+\phi)$, where $\phi=\theta-\frac{\pi}{2}$ and θ is uniformly distributed over $(0,2\pi)$. Verify $|R_{XY}(\tau)|\le\sqrt{R_{XX}(0)R_{YY}(0)}$. (8)

(ii) Find the Power spectral density of a random binary transmission process where autocorrelation function is $R(\tau)=\left\{1-\frac{|\tau|}{T}:|\tau|\le T\right.$. (8)

Or

(b) (i) If the power spectral density of a continuous process is $S_{XX}(\omega)=\frac{\omega^2+9}{\omega^4+5\omega^2+4}$, find the mean square value of the process. (8)

(ii) A stationary process has an autocorrelation function given by $R(\tau)=\frac{25\tau^2+36}{6.25\tau^2+4}$. Find the mean value, mean-square value and variance of the process. (8)

15. (a) (i) If the input to a time invariant stable linear system is a wide sense stationary process, prove that the output will also be a wide sense stationary process. (8)

(ii) Show that $S_{YY}(\omega)=|H(\omega)|^2 S_{XX}(\omega)$ where $S_{XX}(\omega)$ and $S_{YY}(\omega)$ are the power spectral densities of the input $X(t)$ and output $y(t)$ respectively and $H(\omega)$ is the system transfer function. (8)

Or

(b) (i) A circuit has an impulse response given by $h(t)=\begin{cases}\frac{1}{T} : 0 \le t \le T\end{cases}$. Express $S_{YY}(\omega)$ in terms of $S_{XX}(\omega)$. (8)

(ii) Given $R_{XX}(\tau)=Ae^{-\alpha|\tau|}$ and $h(t)=e^{-\beta t}u(t)$ where $u(t)=\begin{cases}1 : t \ge 0 \\ 0 : \text{otherwise}\end{cases}$. Find the spectral density of the output $Y(t)$. (8)
